Implementation of qutrit-based quantum information processing via state-dependent forces on trapped ions

Li-Xiang Cen,[1] Bang-Pin Hou,[2] and Ming-Lun Chen[1]

[1]*Department of Physics, Sichuan University, Chengdu 610065, China*
[2]*Department of Physics, Sichuan Normal University, Chengdu 610068, China*

We propose a scheme to realize quantum logic and entanglement for qutrit systems via state-dependent forces on trapped ions. By exploiting the laser-ion coupling in the presence of Coulomb interactions, the set of quantum gate operations including the conditional phase shifts on two qutrits as well as arbitrary SU(3) rotations on single qutrits are derived for universal quantum manipulation. As an illustration, we demonstrate in detail how these gate resources could be used to generate the maximally entangled state of two qutrits. Besides being insensitive to vibrational heating of the trapped ions, the present scheme is also shown to be scalable through designing appropriately the pulse configuration of the laser-ion interactions.

PACS numbers: 03.67.Pp, 03.67.Lx, 03.65.Vf

Quantum information processing based on trapped ions has witnessed rapid development in the past decade. Since the first gate scheme proposed by Cirac and Zoller [1], several scenarios have been proposed for entangling trapped ion quantum bits for quantum information applications [2, 3, 4, 5, 6, 7, 8, 9]. It was shown recently [4, 6, 7, 8, 9] that quantum manipulation could be implemented via state-dependent forces acting on ions in the presence of mutual Coulomb interactions. Experimental studies exploiting optical Raman fields to achieve strong state-dependent forces through ac-Stark shifts have also been reported [10, 11]. Remarkably, owing to the global property of the evolution in the position-momentum phase space, quantum operations based on this kind of laser-ion coupling are insensitive to vibrational heating from noisy background electric fields, therefore relax the physical constraints on implementing quantum information processing.

The conventional building blocks for quantum processors are qubits, i.e., the two-level physical systems. On the other hand, the use of quantum entanglement in high-dimensional quantum systems has also been considered to generalize and improve the qubit-based quantum algorithms and protocols [12]. It was proven that utilization of qutrit systems instead of qubits could be more secure against a symmetric attack on a quantum key distribution protocol [13]. Furthermore, quantum computing power based on qudit quantum processors has been investigated recently and the qutrit-based quantum information processing is found to optimize the Hilbert-space dimensionality hence maximize the computing power [14]. Motivated by these facts, designs of quantum manipulation on qutrit systems including the direct extension of the original Cirac-Zoller scenario have also been investigated [15, 16].

In this paper we shall present a scheme to realize the qutrit-based quantum manipulation by making use of the state-dependent laser-ion coupling. The derived set of quantum logical operations, including the conditional phase shifts on two qutrits and arbitrary SU(3) rotations on single qutrits, promises the universal processing of quantum information. As an illustration, the maximally entangled state of two qutrits is consequently generated by using these gate resources. The built scheme of quantum manipulation is believed to be insensitive to vibrational temperature of ions. Moreover, the potential to scale up the ion array system is also demonstrated through designing appropriately the pulse configuration of the laser-ion interactions.

Let us consider a system of N ions confined in a one-dimensional linear Paul trap with a global trap frequency ω. The hyperfine internal states of each ion, $\{|0\rangle, |1\rangle, |2\rangle\}$, are selected to encode information for qutrits. In our scheme, the rotations between any two levels of single qutrits, namely, the transformations $R_{mm'}(\theta,\phi)$ specified by

$$\begin{aligned} R_{mm'}(\theta,\phi)|m\rangle &= \cos\theta|m\rangle + ie^{i\phi}\sin\theta|m'\rangle, \\ R_{mm'}(\theta,\phi)|m'\rangle &= ie^{-i\phi}\sin\theta|m\rangle + \cos\theta|m'\rangle \end{aligned} \quad (1)$$

with $\{|m\rangle, |m'\rangle\}$ denoting any one of the sets $\{|0\rangle, |1\rangle\}$, $\{|0\rangle, |2\rangle\}$ and $\{|1\rangle, |2\rangle\}$, are performed without involving the motional degree of freedom of ions. This is attainable in physics either by applying resonant microwave fields to induce coherent Rabi oscillations between the states $|m\rangle$ and $|m'\rangle$, or by exploiting optical Raman fields to generate the stimulated-Raman transitions through coupling virtually the excited states [17]. Note that the rotations (1) account for a series of SU(2) transformations that form the necessity ingredients of the general SU(3) operation for single qutrits.

The central issue we are going to discuss is the implementation of conditional logic for two qutrits, in which the role of the vibrational mode is necessary as it effectively couples the ions together in the presence of Coulomb interactions. In the absence of external forces, the ion motion is well characterized by the collective harmonic oscillators under the second-order expansion of the potential for small vibrations. For concreteness, we will focus on quantum operations on $N = 2$ ions, and leave the extension to large number of ions for a late discus-

sion. The base Hamiltonian for the system is therefore described as (setting $\hbar = 1$)

$$H_0 = \sum_{k=c,r} \omega_k a_k^\dagger a_k + \sum_{\mu=1,2} [\Delta_1 \sigma_{11}^{(\mu)} + \Delta_2 \sigma_{22}^{(\mu)}], \qquad (2)$$

where $\omega_c = \omega$ and ω_r denote the frequencies associated with the center-of-mass and stretch modes, respectively; and $a_k(a_k^\dagger)$ are their corresponding annihilation (creation) operators. The notation $\sigma_{mm'}^{(\mu)} = |m\rangle\langle m'|$ $(m, m' = 0, 1, 2)$ accounts for internal level operators of the ion μ, and Δ_1 and Δ_2 are energy differences between the levels $|0\rangle \leftrightarrow |1\rangle$ and $|1\rangle \leftrightarrow |2\rangle$, respectively. To perform quantum gates, we assume that the ions could be addressed individually by laser beams and the acceleration force is hence created depending on the ion internal state, namely, $F_\mu(t) = \sum_m f_{\mu,m}(t)\sigma_{mm}^{(\mu)}$ with coefficients $f_{\mu,m}(t)$ specified by the detailed laser-ion coupling [18]. The related interaction term is given by $H_F^{(\mu)}(t) = -x_\mu F_\mu(t)$, where the local coordinator x_μ $(\mu = 1, 2)$ relates to the collective one by $q_c = (x_1 + x_2)/\sqrt{2}$ and $q_r = (x_1 - x_2)/\sqrt{2}$. As the two ions are exerted by the external acceleration forces simultaneously, the Hamiltonian of the system in the rotation frame with respect to H_0 takes the following general form

$$H_I(t) = -\sum_{k,m} [g_{1,m}^k(t)\sigma_{mm}^{(1)} + g_{2,m}^k(t)\sigma_{mm}^{(2)}] a_k e^{-i\omega_k t} + h.c.. \qquad (3)$$

Here, the sums are taken over $k = c, r$ and $m = 0, 1, 2$, and the coefficients are given by $g_{\mu,m}^k(t) = D_{\mu k} f_{\mu,m}(t)/\sqrt{2M\omega_k}$ in which D is an orthogonal matrix $D = \frac{1}{\sqrt{2}}\left(\begin{smallmatrix} 1 & 1 \\ 1 & -1 \end{smallmatrix}\right)$ and M denotes the mass of the ion.

To explore the time evolution of the system, we resort to a gauged representation [19] with respect to the unitary transformation $G(t) = \exp[-i\int_0^t H_I(\tau)d\tau]$. It is directly shown that the time evolution operator of the system reads $U(t) = G(t)U_g(t)$, where $U_g(t)$ satisfies the covariant equation $i\partial_t U_g(t) = H_I^g(t)U_g(t)$ and the gauged Hamiltonian $H_I^g(t)$ is worked out to be

$$\begin{aligned} H_I^g(t) &= G^{-1}H_I G - iG^{-1}\partial G/\partial t \\ &= \sum_{m,n} J_{mn}(t)\sigma_{mm}^{(1)}\sigma_{nn}^{(2)} \\ &\quad + \sum_m [\epsilon_{1,m}(t)\sigma_{mm}^{(1)} + \epsilon_{2,m}(t)\sigma_{mm}^{(2)}], \end{aligned} \qquad (4)$$

where the coefficients $J_{mn}(t)$ and $\epsilon_{\mu,m}(t)$ $(\mu = 1, 2)$ are given by

$$\begin{aligned} J_{mn}(t) &= \sum_{k=c,r} \int_0^t [g_{1,m}^k(t)g_{2,n}^k(t') + g_{1,m}^k(t')g_{2,n}^k(t)] \\ &\quad \times \sin\omega_k(t' - t)dt', \\ \epsilon_{\mu,m}(t) &= \sum_{k=c,r} \int_0^t g_{\mu,m}^k(t)g_{\mu,m}^k(t')\sin\omega_k(t' - t)dt'. \end{aligned} \qquad (5)$$

It is seen that the pure qutrit-qutrit coupling is explicitly manifested in the gauged Hamiltonian of Eq. (4), and the coupling of the qutrit with phonon degree of freedom is only indicated by the transformation $G(t)$. Especially, as the external force is controlled with a particular configuration such that

$$\int_0^T g_{\mu,m}^k(t)e^{-i\omega_k t}dt = 0, \quad k = c, r, \qquad (6)$$

the transformation $G(T)$ becomes an identity operator. Consequently, the evolution operator of the system at time T is exactly obtained as

$$\begin{aligned} U(T) &= U_g(T) \\ &= \left[\prod_{m,n=0}^{2} P_{mn}\right]\left[\prod_{m=0}^{2} D_m^{(1)}\right]\left[\prod_{m=0}^{2} D_m^{(2)}\right], \end{aligned} \qquad (7)$$

where

$$P_{mn} \equiv e^{-i\Phi_{mn}(T)\sigma_{mm}^{(1)}\sigma_{nn}^{(2)}}, \quad D_m^{(\mu)} \equiv e^{-i\phi_{\mu,m}(T)\sigma_{mm}^{(\mu)}}, \qquad (8)$$

and the coefficients are given by

$$\Phi_{mn}(T) = \int_0^T J_{mn}(t)dt, \phi_{\mu,m}(T) = \int_0^T \epsilon_{\mu,m}(t)dt. \qquad (9)$$

The intriguing feature of the above process is that the generated overall evolution (7) contains no operator entangling the qutrit with vibrational degrees of freedom, hence is in sense global and fault tolerant against the vibrational heating of the ions. The first factor contained in the expression of (7) stands explicitly for a general phase shift operation for two qutrits. The derived form is actually universal in view that it exhausts all of the possible two-qutrit logic consisting of phase flip $P_{mn}(\Phi)$, providing that appropriate selections of the laser-ion coupling are promised. It is seen that the induced transformation (7) includes also two extra factors composed of operators $D_m^{(\mu)}$ acting on the two qutrits individually. Differing from the case of the qubit system, these extra operations are inevitable in the evolution specified by Eqs. (4) and (5) and indicate excessive actions in the above process to achieve the desired two-qutrit phase shift operation. To remove these undesirable actions, we present in the following a scheme to achieve the independent phase shift operation for single qutrits which combining with the transformation (7) could induce the pure logical operations for two qutrits.

In detail, we exploit the same physical setup to exert the acceleration force on the individual ion μ, by which the Hamiltonian in the rotation frame reads

$$H_I^{(\mu)}(t) = -\sum_{k=c,r} [g_m^k(t)a_k e^{-i\omega_k t} + h.c.]\sigma_{mm}^{(\mu)}, \qquad (10)$$

where $m = 0, 1,$ or 2. As the system undergoes a cyclic evolution such that $\int_0^T g_m^k(t)e^{-i\omega_k t}dt = 0$, the evolution operator of the system is obtained as

$$U(T) = e^{-i\phi(T)\sigma_{mm}^{(\mu)}} \equiv D_m^{(\mu)}(\phi), \qquad (11)$$

where

$$\phi(T) = \sum_{k=c,r} \int_0^T \int_0^t g_m^k(t) g_m^k(t') \sin\omega_k(t'-t) dt' dt. \quad (12)$$

Physically, the ion μ is displaced coherently in the evolution if the electron inhabits in the level $|m\rangle$. By designing the cyclic force configuration such that the ion returns to its original state in the phase-momentum space, the vibrational degree of freedom will become disentangled with the qutrit and an effective phase shift independent of the motional state hence is induced for the ion internal state. Note that there is the relation $\sum_m \sigma_{mm}^{(\mu)} \equiv I$, hence for each ion only two of the operators in (11) are independent. The significance of the phase shift operations (11) derived for single qutrits is actually multifold. Firstly, by noticing that all the operators $D_m^{(1)}$, $D_m^{(2)}$, and P_{mn} are commutative with each other, the independent manipulation of $D_m^{(\mu)}(\phi)$ therefore provides a distinct way to remove the extra operations in (7) so that the pure conditional phase gate for two qutrits could be achieved. Moreover, it is noteworthy that the two independent phase gates of (11) enable the scheme, together with the series of SU(2) rotations indicated in Eq. (1), to perform universal rotations for single qutrits. Finally, we point out that the state evolution described above is expressed in the rotation frame. It differs from the original Schrodinger picture by a transformation e^{-iH_0T} which would induce relative phase accumulations to the qutrit states. With the help of the phase-shift operation (11), the accumulation of additional phases could be in principle canceled out.

Up to now, we have shown that the universal rotation for single qutrits as well as the two-qutrit phase shift operation could be achieved by virtue of the state-dependent forces on trapped ions. These gate resources are sufficient to generate universal quantum manipulations for qutrit systems. In particular, we show in the following the utilization of the proposed gate operations to achieve the maximally entangled state for two qutrits. In detail, assume that the two qutrits are initially prepared in a state $|0\rangle \otimes |0\rangle$. We first exert the same rotation on each qutrit individually to derive a state $|\psi\rangle = |+\rangle \otimes |+\rangle$, where

$$\begin{aligned} |+\rangle &= R_{12}(\pi/4, -\pi/2) R_{01}(\arctan\sqrt{2}, -\pi/2)|0\rangle \\ &= \frac{1}{\sqrt{3}}(|0\rangle + |1\rangle + |2\rangle). \end{aligned} \quad (13)$$

Then by applying the sequence of two-qutrit phase flip operations we obtain

$$\begin{aligned} |\psi_M'\rangle &= P_{33}(2\pi/3) P_{22}(2\pi/3) P_{11}(2\pi/3)|\psi\rangle \\ &= \frac{1}{\sqrt{3}}(|00'\rangle + |11'\rangle + |22'\rangle), \end{aligned} \quad (14)$$

where

$$\begin{aligned} |0'\rangle &= \frac{1}{\sqrt{3}}(e^{-i2\pi/3}|0\rangle + |1\rangle + |2\rangle), \\ |1'\rangle &= \frac{1}{\sqrt{3}}(|0\rangle + e^{-i2\pi/3}|1\rangle + |2\rangle), \\ |2'\rangle &= \frac{1}{\sqrt{3}}(|0\rangle + |1\rangle + e^{-i2\pi/3}|2\rangle). \end{aligned} \quad (15)$$

In Eq. (14) we have employed the aforementioned design using joint evolution indicated by Eqs. (7) and (11) to achieve the pure phase shift operations $P_{mm}(2\pi/3)$ with $m = 1, 2, 3$. Consequently, the maximally entangled state in the conventional computational basis, i.e., $|\psi_M\rangle = \frac{1}{\sqrt{3}} \sum_{m=0}^{2} |mm\rangle$, could be obtained by (up to an irrelevant global phase)

$$\begin{aligned} & R_{01}(\pi/4, 5\pi/6) R_{02}(\arctan 1/\sqrt{2}, 5\pi/6) \\ & \times R_{12}(\pi/4, \pi) D_1(\pi/6) |\psi_M'\rangle \to |\psi_M\rangle, \end{aligned} \quad (16)$$

where the sequence of operations $R_{01}R_{02}R_{12}D_1$ are performed on arbitrary one of the two qutrits.

The scheme we discussed till now is restricted to the system of two ions. The extended version of the system with large N ions should be described by a Hamiltonian of form (3) where in stead the sum of phonon modes takes over $k = 1, \cdots, N$ and the coefficients are given by $g_{\mu,m}^k = D_{\mu k} f_{\mu,m}/\sqrt{2M\omega_k}$ with D the unitary transformation diagonalising the Hessian matrix. Note that the commensurability condition (6) is now replaced by the following generalized one

$$\int_0^T g_{\mu,m}^k(t) e^{-i\omega_k t} dt = 0, k = 1, \cdots, N \quad (17)$$

which becomes very fragile due to the increased complexity of phonon mode spectrum. One possible resolution to this difficulty is to carry out the acceleration forces by an adiabatic manner. Specifically, suppose that $f_{\mu,m}(t)$ describing the configuration of the pushing force is some smooth function and satisfies $|\dot{f}_{\mu,m}(t)|/\sqrt{2M\omega_c} \ll \omega_c$ where $\omega_c = \omega$ is the frequency of the center-of-mass mode, i.e., the lowest frequency amongst all of the phonon modes ω_k. If the coefficient $g_{\mu,m}^k(t)$ which is proportional to $f_{\mu,m}(t)$ undergoes from 0 to some finite value and then back to 0 as time goes from 0 to T, then all the relations of (17) come into existence

$$\int_0^T g_{\mu,m}^k(t) e^{-i\omega_k t} dt = \int_0^T i[\dot{g}_{\mu,m}^k(t)/\omega_k] e^{-i\omega_k t} dt \to 0. \quad (18)$$

Moreover, the phases in Eq. (9) for this special case are worked out to be

$$\begin{aligned} \Phi_{mn}(T) &= -2 \sum_{k=1}^{N} \int_0^T g_{1,m}^k(t) g_{2,n}^k(t)/\omega_k dt, \\ \phi_{\mu,m}(T) &= -\sum_{k=1}^{N} \int_0^T [g_{\mu,m}^k(t)]^2/\omega_k dt. \end{aligned} \quad (19)$$

The intrinsic drawback associated with the above adiabatic scenario is the slow evolution of the gate operation

which gives decoherence more time to exert its detrimental effects. An alternative scaling scenario is to employ the fast gate scheme combining with a noise cancelation strategy of refocusing techniques [7, 20]. Briefly, the key point of the strategy is to observe that the opposite loop evolution generated by reversal force configuration, say, $f^c_{\mu,m}(t)$ and $f^{\bar{c}}_{\mu,m}(t) = -f^c_{\mu,m}(t)$, would lead to the same $U_g(T)$ but reversed ingredients of the noise contribution: $G_{\bar{c}}(T) = G^{-1}_c(T)$. Therefore the influence of non-vanishing $G(T)$ could be effectively suppressed by refocusing the gate pulses providing that the periodic laser pulse is faster than all the frequencies of the vibrational modes. For the further demonstration of validity of the scheme including numerical illustration of the gate infidelity in the presence of thermal phonon excitations, as the detailed discussion for ion qubits was presented in Ref. [7], we point out that the same argument is valid for the present scenario of extended qutrit systems.

In summary, we have proposed a scheme to implement the qutrit-based quantum information processing by making use of the state-dependent laser-ion coupling. The derived quantum logical operations are believed to possess the noise resilience property against vibrational heating of ions owing to the global feature of the evolution in the phase space. We present also a detailed illustration to generate the maximally entangled state of two qutrits by utilizing these gate resources. Finally, the potential to scale up the gate scheme for large-scale information processing is also demonstrated, either through designing an adiabatic way of switching on and off the interactions, or by a fast gate scenario with refocusing strategy to cancel out the unwanted noise influence.

Support from the Youth Science Research Grants of Sichuan University is gratefully acknowledged.

[1] J.I. Cirac and P. Zoller, Phys. Rev. Lett. **74**, 4091 (1995).

[2] A. Sorensen and K. Molmer, Phys. Rev. Lett. **82**, 1971 (1999); A. Sorensen and K. Molmer, Phys. Rev. A **62**, 022311 (2000); G.J. Milburn, S. Schneider, and D.F.V. James, Fortschr. Phys. **48**, 801 (2000).

[3] D. Jonathan, M.B. Plenio, and P.L. Knight, Phys. Rev. A **62**, 042307 (2000).

[4] J.I. Cirac and P. Zoller, Nature **404**, 579 (2000).

[5] L.F. Wei, S.Y. Liu, and X.L. Lei, Phys. Rev. A **65**, 062316 (2002)

[6] JJ. Garcia-Ripoll, P. Zoller, and J.I. Cirac, Phys. Rev. Lett. **91**, 157901 (2003).

[7] L.-M. Duan, Phys. Rev. Lett. **93**, 100502 (2004).

[8] S.L. Zhu, C. Monroe, and L.M. Duan, Europhys. Lett. **73**, 485 (2006).

[9] L.-X. Cen, Z.D. Wang, and S.J. Wang, quant-ph/0603222.

[10] D. Leibfried *et al.*, Nature **422**, 412 (2003).

[11] P.C. Haljan, K.-A. Brickman, L. Deslauriers, P.J. Lee, and C. Monroe, Phys. Rev. Lett. **94**, 153602 (2005).

[12] C.M. Caves and G.J. Milburn, Opt. Commun. **179**, 439 (2000).

[13] D. bruss and C. Macchiavello, Phys. Rev. Lett. **88**, 127901 (2002).

[14] A.D. Greentree etc., Phys. Rev. Lett. **92**, 097901 (2004).

[15] A.B. Klimov, R. Guzman, J.C. Retamal, and C. Saavedra, Phys. Rev. A **67**, 062313 (2003).

[16] D.M. Hugh and J. Twamley, quant-ph/0506031.

[17] D. Leibfried et al., Rev. Mod. Phys. **75**, 281 (2003).

[18] D.J. Wineland et al., quant-ph/0505203.

[19] S.J. Wang, F.L. Li and A. Weiguny, Phys. Lett. A **180**, 189 (1993); L.-X. Cen, X.Q. Li, Y.J. Yan, H.Z. Zheng, and S.J. Wang, Phys. Rev. Lett. **90**, 147902 (2003).

[20] L.-X. Cen and P. Zanardi, Phys. Rev. A **71**, R060307 (2005).